

1. Bestimme zu folgenden Bruchgleichungen die Definitionsmenge.

a) $\frac{x}{x-5} = \frac{7}{8}$

b) $\frac{1}{x} = \frac{x-3}{x+2}$

c) $\frac{4-x}{6-x} = \frac{5+x}{x+9}$

d) $\frac{2x-1}{1+3x} = \frac{1+2x}{2-7x}$



2. Löse die Bruchgleichungen und gib den Definitionsbereich an.

a) $\frac{1}{x-3} = 1$

b) $\frac{9}{x-1} = 3$

c) $\frac{1}{x+5} = 1$

d) $8 - \frac{14}{x} = \frac{10}{x}$

e) $\frac{9}{x} = \frac{2}{x} + \frac{7}{8}$

f) $\frac{5}{2x} + \frac{7}{4x} = 2$

g) $\frac{4}{3x} - 1 = \frac{6}{5x}$

h) $\frac{3}{x-2} = \frac{2}{x-3}$

i) $\frac{3}{2x-1} = \frac{2}{x+3}$

j) $\frac{3}{2x-1} = \frac{7}{4x-2}$

3. Löse die Bruchgleichungen und gib den Definitionsbereich an.

$\frac{1}{2x-4} = \frac{1}{3x-6}$



4. a) $\frac{12}{x+4} = 1$

b) $\frac{15}{x-3} = 3$

c) $\frac{12}{x+1} + 2 = 4$

d) $\frac{42}{x-2} - 3 = 4$

e) $\frac{12}{x+5} + 3 = 4$

f) $3 - \frac{3}{x-2} = 2$

5. a) $\frac{5}{2x} + 1 = \frac{3}{x}$

b) $\frac{6}{x} + 1 = \frac{9}{x}$

c) $\frac{2}{3x} + \frac{1}{2x} + 1 = \frac{1}{6x}$

d) $\frac{4}{10x} + 1 = \frac{9}{10x} - \frac{1}{5x}$

e) $\frac{3}{2x} - \frac{2}{3} = \frac{5}{6x}$

f) $\frac{3}{4x} - \frac{1}{6x} = \frac{2}{3}$

6. a) $\frac{3}{x-3} = \frac{5}{x-5}$

b) $\frac{4}{x-2} = \frac{6}{x+1}$

c) $\frac{5}{x+2} = \frac{3}{x-2}$

d) $\frac{5}{x+1} = \frac{2}{x-2}$

e) $\frac{4}{x+3} = \frac{6}{x+5}$

f) $\frac{8}{x} = \frac{5}{x-3}$

7. a) $\frac{x-5}{x+3} = \frac{x-6}{x-2}$

b) $\frac{4x+7}{3x+4} = \frac{8x+11}{6x+6}$

c) $\frac{3x+14}{2x-4} = \frac{3x+2}{2x-7}$

d) $\frac{x+1}{x-1} = \frac{6x+11}{6x-3}$

e) $\frac{x+1}{x+4} = \frac{x-2}{x+1}$

f) $\frac{3x+6}{4x+8} = \frac{3x+16}{4x+33}$



Lösung 1:

- a) $\mathbb{Q}\{5\}$
 b) $\mathbb{Q}\{-2; 0\}$
 c) $\mathbb{Q}\{-9; 6\}$
 d) $\mathbb{Q}\{-\frac{1}{3}; \frac{2}{7}\}$

Lösung 2:

- a) $x = 4; D = \mathbb{Q}\{3\}$
 b) $x = 4; D = \mathbb{Q}\{1\}$
 c) $x = -4; D = \mathbb{Q}\{-5\}$
 d) $x = 3; D = \mathbb{Q}\{0\}$
 e) $x = 8; D = \mathbb{Q}\{0\}$
 f) $x = \frac{17}{8}; D = \mathbb{Q}\{0\}$
 g) $x = \frac{2}{15}; D = \mathbb{Q}\{0\}$
 h) $x = 5; D = \mathbb{Q}\{2; 3\}$
 i) $x = 11; D = \mathbb{Q}\{0,5; -3\}$
 j) keine Lösung; $D = \mathbb{Q}\{0,5\}$

Lösung 3:

Die Bruchgleichung hat keine Lösung. Umformungen der Bruchgleichung führen zwar auf $x = 2$, es gilt aber die Einschränkung $x \neq 2$.

4. a) $\frac{12}{x+4} = 1$
 $D = \mathbb{Q} \setminus \{-4\}$
 HN: $x + 4$
 $12 = x + 4$
 $L = \{8\}$
- b) $\frac{15}{x-3} = 3$
 $D = \mathbb{Q} \setminus \{3\}$
 HN: $x - 3$
 $15 = 3x - 9$
 $L = \{8\}$
- c) $\frac{12}{x+1} + 2 = 4$
 $D = \mathbb{Q} \setminus \{-1\}$
 HN: $x + 1$
 $12 = 2x + 2$
 $L = \{5\}$
- d) $\frac{42}{x-2} - 3 = 4$
 $D = \mathbb{Q} \setminus \{2\}$
 HN: $x - 2$
 $42 = 7x - 14$
 $L = \{8\}$
- e) $\frac{12}{x+5} + 3 = 4$
 $D = \mathbb{Q} \setminus \{-5\}$
 HN: $x + 5$
 $12 = x + 5$
 $L = \{7\}$
- f) $3 - \frac{3}{x-2} = 2$
 $D = \mathbb{Q} \setminus \{2\}$
 HN: $x - 2$
 $3 = x - 2$
 $L = \{5\}$
5. a) $\frac{5}{2x} + 1 = \frac{3}{x}$
 $D = \mathbb{Q} \setminus \{0\}$
 HN: $2x$
 $5 + 2x = 6$
 $L = \{0,5\}$
- b) $\frac{6}{x} + 1 = \frac{9}{x}$
 $D = \mathbb{Q} \setminus \{0\}$
 HN: x
 $6 + x = 9$
 $L = \{3\}$
- c) $\frac{2}{3x} + \frac{1}{2x} + 1 = \frac{1}{6x}$
 $D = \mathbb{Q} \setminus \{0\}$
 HN: $6x$
 $4 + 3 + 6x = 1$
 $L = \{-1\}$
- d) $\frac{4}{10x} + 1 = \frac{9}{10x} - \frac{1}{5x}$
 $D = \mathbb{Q} \setminus \{0\}$
 HN: $10x$
 $4 + 10x = 9 - 2$
 $L = \{0,3\}$
- e) $\frac{3}{2x} - \frac{2}{3} = \frac{5}{6x}$
 $D = \mathbb{Q} \setminus \{0\}$
 HN: $6x$
 $9 - 4x = 5$
 $L = \{1\}$
- f) $\frac{3}{4x} - \frac{1}{6x} = \frac{2}{3}$
 $D = \mathbb{Q} \setminus \{0\}$
 HN: $12x$
 $9 - 2 = 8x$
 $L = \left\{ \frac{7}{8} \right\}$

6.

a) $\frac{3}{x-3} = \frac{5}{x-5}$

$D = \mathbb{Q} \setminus \{3; 5\}$

HN: $(x-3)(x-5)$

$3x - 15 = 5x - 15$

$L = \{0\}$

b) $\frac{4}{x-2} = \frac{6}{x+1}$

$D = \mathbb{Q} \setminus \{2; -1\}$

HN: $(x-2)(x+1)$

$4x + 4 = 6x - 12$

$L = \{8\}$

c) $\frac{5}{x+2} = \frac{3}{x-2}$

$D = \mathbb{Q} \setminus \{2; -2\}$

HN: $(x+2)(x-2)$

$5x - 10 = 3x + 6$

$L = \{8\}$

d) $\frac{5}{x+1} = \frac{2}{x-2}$

$D = \mathbb{Q} \setminus \{-1; 2\}$

HN: $(x+1)(x-2)$

$5x - 10 = 2x + 2$

$L = \{4\}$

e) $\frac{4}{x+3} = \frac{6}{x+5}$

$D = \mathbb{Q} \setminus \{-5; -3\}$

HN: $(x+3)(x+5)$

$4x + 20 = 6x + 18$

$L = \{1\}$

f) $\frac{8}{x} = \frac{5}{x-3}$

$D = \mathbb{Q} \setminus \{0; 3\}$

HN: $x(x-3)$

$8x - 24 = 5x$

$L = \{8\}$

7.

a) $\frac{x-5}{x+3} = \frac{x-6}{x-2}$

$D = \mathbb{Q} \setminus \{-3; 2\}$

HN: $(x+3)(x-2)$

$(x-5)(x-2) = (x-6)(x+3)$

$L = \{7\}$

b) $\frac{4x+7}{3x+4} = \frac{8x+11}{6x+6}$

$D = \mathbb{Q} \setminus \{-\frac{4}{3}; -1\}$

HN: $(3x+4)(6x+6)$

$(4x+7)(6x+6) = (8x+11)(3x+4)$

$L = \{2\}$

c) $\frac{3x+14}{2x-4} = \frac{3x+2}{2x-7}$

$D = \mathbb{Q} \setminus \{2; \frac{7}{2}\}$

HN: $(2x-4)(2x-7)$

$(3x+14)(2x-7) = (3x+2)(2x-4)$

$L = \{6\}$

d) $\frac{x+1}{x-1} = \frac{6x+11}{6x-3}$

$D = \mathbb{Q} \setminus \{1; \frac{1}{2}\}$

HN: $(x-1)(6x-3)$

$(x+1)(6x-3) = (6x+11)(x-1)$

$L = \{4\}$

e) $\frac{x+1}{x+4} = \frac{x-2}{x+1}$

$D = \mathbb{Q} \setminus \{-4; -1\}$

HN: $(x+4)(x+1)$

$(x+4)(x+1) = (x-2)(x+4)$

$1 = -8$

$L = \{ \}$

f) $\frac{3x+6}{4x+8} = \frac{3x+16}{4x+33}$

$D = \mathbb{Q} \setminus \{-2; -8,25\}$

HN: $(4x+8)(4x+33)$

$(3x+6)(4x+33) = (3x+16)(4x+8)$

$16 = \frac{99}{4}$

$L = \{ \}$